Attia, John Okyere. “Fourier Analysis.”
Electronics and Circuit Analysis using MATLAB.
Ed. John Okyere Attia
Boca Raton: CRC Press LLC, 1999

CHAPTER EIGHT

FOURIER ANALYSIS

In this chapter, Fourier analysis will be discussed. Topics covered are Fourier series expansion, Fourier transform, discrete Fourier transform, and fast Fourier transform. Some applications of Fourier analysis, using MATLAB, will also be discussed.

8.1 FOURIER SERIES

If a function $g(t)$ is periodic with period T_p, i.e.,

$$g(t) = g(t \pm T_p) \tag{8.1}$$

and in any finite interval $g(t)$ has at most a finite number of discontinuities and a finite number of maxima and minima (Dirichlets conditions), and in addition,

$$\int_0^{T_p} g(t)dt < \infty \tag{8.2}$$

then $g(t)$ can be expressed with series of sinusoids. That is,

$$g(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nw_0 t) + b_n \sin(nw_0 t) \tag{8.3}$$

where

$$w_0 = \frac{2\pi}{T_p} \tag{8.4}$$

and the Fourier coefficients a_n and b_n are determined by the following equations.

$$a_n = \frac{2}{T_p} \int_{t_o}^{t_o+T_p} g(t)\cos(nw_0 t)dt \qquad n = 0, 1,2, \ldots \tag{8.5}$$

$$b_n = \frac{2}{T_p} \int_{t_o}^{t_o+T_p} g(t) \sin(nw_0 t) dt \qquad n = 0, 1, 2 \ldots \tag{8.6}$$

Equation (8.3) is called the trigonometric Fourier series. The term $\frac{a_0}{2}$ in Equation (8.3) is the dc component of the series and is the average value of $g(t)$ over a period. The term $a_n \cos(nw_0 t) + b_n \sin(nw_0 t)$ is called the *n*-th harmonic. The first harmonic is obtained when $n = 1$. The latter is also called the fundamental with the fundamental frequency of ω_o. When n = 2, we have the second harmonic and so on.

Equation (8.3) can be rewritten as

$$g(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} A_n \cos(nw_0 t + \Theta_n) \tag{8.7}$$

where

$$A_n = \sqrt{a_n^2 + b_n^2} \tag{8.8}$$

and

$$\Theta_n = -\tan^{-1}\left(\frac{b_n}{a_n}\right) \tag{8.9}$$

The total power in $g(t)$ is given by the Parseval's equation:

$$P = \frac{1}{T_p} \int_{t_o}^{t_o+T_p} g^2(t) dt = A_{dc}^2 + \sum_{n=1}^{\infty} \frac{A_n^2}{2} \tag{8.10}$$

where

$$A_{dc}^2 = \left(\frac{a_0}{2}\right)^2 \tag{8.11}$$

The following example shows the synthesis of a square wave using Fourier series expansion.

Example 8.1

Using Fourier series expansion, a square wave with a period of 2 ms, peak-to-peak value of 2 volts and average value of zero volt can be expressed as

$$g(t) = \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{(2n-1)}\sin[(2n-1)2\pi f_0 t\ ] \qquad (8.12)$$

where

$$f_0 = 500 \text{ Hz}$$

if $a(t)$ is given as

$$a(t) = \frac{4}{\pi}\sum_{n=1}^{12}\frac{1}{(2n-1)}\sin[(2n-1)2\pi f_0 t\ ] \qquad (8.13)$$

Write a MATLAB program to plot $a(t)$ from 0 to 4 ms at intervals of 0.05 ms and to show that $a(t)$ is a good approximation of *g(t)*.

Solution

MATLAB Script

```
% fourier series expansion
f = 500; c = 4/pi; dt = 5.0e-05;
tpts = (4.0e-3/5.0e-5) + 1;
for n = 1: 12
for m = 1: tpts
s1(n,m) = (4/pi)*(1/(2*n - 1))*sin((2*n - 1)*2*pi*f*dt*(m-1));
end
end
for m = 1:tpts
 a1 = s1(:,m);
 a2(m) = sum(a1);
end
f1 = a2';
t = 0.0:5.0e-5:4.0e-3;
clg
plot(t,f1)
xlabel('Time, s')
```

```
ylabel('Amplitude, V')
title('Fourier series expansion')
```

Figure 8.1 shows the plot of $a(t)$.

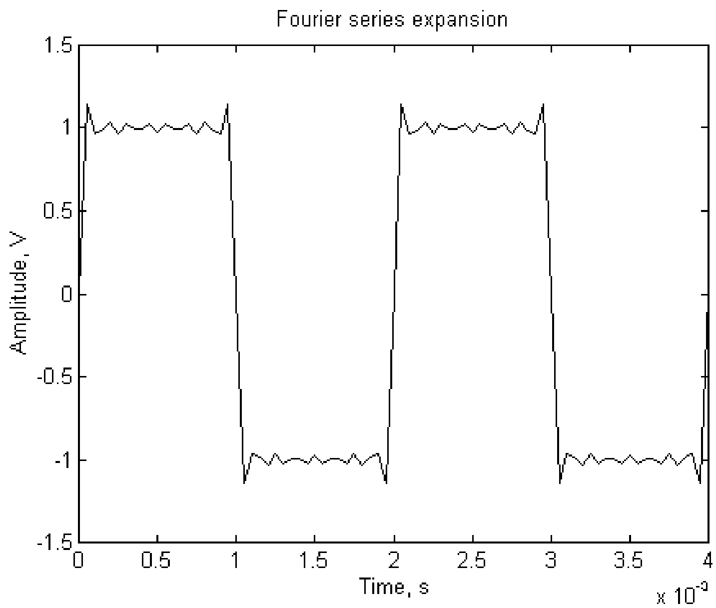

Figure 8.1 Approximation to Square Wave

By using the Euler's identity, the cosine and sine functions of Equation (8.3) can be replaced by exponential equivalents, yielding the expression

$$g(t) = \sum_{n=-\infty}^{\infty} c_n \exp(jnw_0 t) \tag{8.14}$$

where

$$c_n = \frac{1}{T_p} \int_{-t_p/2}^{T_p/2} g(t) \exp(-jnw_0 t) dt \tag{8.15}$$

and

$$w_0 = \frac{2\pi}{T_p}$$

Equation (8.14) is termed the exponential Fourier series expansion. The coefficient c_n is related to the coefficients a_n and b_n of Equations (8.5) and (8.6) by the expression

$$c_n = \frac{1}{2}\sqrt{a_n^2 + b_n^2}\angle - \tan^{-1}(\frac{b_n}{a_n}) \tag{8.16}$$

In addition, c_n relates to A_n and ϕ_n of Equations (8.8) and (8.9) by the relation

$$c_n = \frac{A_n}{2}\angle\Theta_n \tag{8.17}$$

The plot of $|c_n|$ versus frequency is termed the discrete amplitude spectrum or the line spectrum. It provides information on the amplitude spectral components of $g(t)$. A similar plot of $\angle c_n$ versus frequency is called the discrete phase spectrum and the latter gives information on the phase components with respect to the frequency of $g(t)$.

If an input signal $x_n(t)$

$$x_n(t) = c_n \exp(jnw_o t) \tag{8.18}$$

passes through a system with transfer function $H(w)$, then the output of the system $y_n(t)$ is

$$y_n(t) = H(jnw_o)c_n \exp(jnw_o t) \tag{8.19}$$

The block diagram of the input/output relation is shown in Figure 8.2.

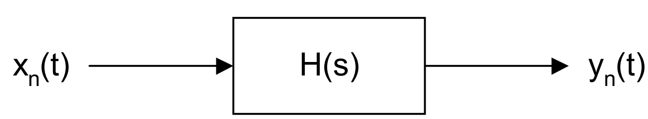

Figure 8.2 Input/Output Relationship

However, with an input $x(t)$ consisting of a linear combination of complex excitations,

$$x_n(t) = \sum_{n=-\infty}^{\infty} c_n \exp(jnw_o t) \tag{8.20}$$

the response at the output of the system is

$$y_n(t) = \sum_{n=-\infty}^{\infty} H(jnw_o) c_n \exp(jnw_o t) \tag{8.21}$$

The following two examples show how to use MATLAB to obtain the coefficients of Fourier series expansion.

Example 8.2

For the full-wave rectifier waveform shown in Figure 8.3, the period is 0.0333s and the amplitude is 169.71 Volts.

(a) Write a MATLAB program to obtain the exponential Fourier series coefficients c_n for $n =$ 0,1, 2, .. , 19

(b) Find the dc value.

(c) Plot the amplitude and phase spectrum.

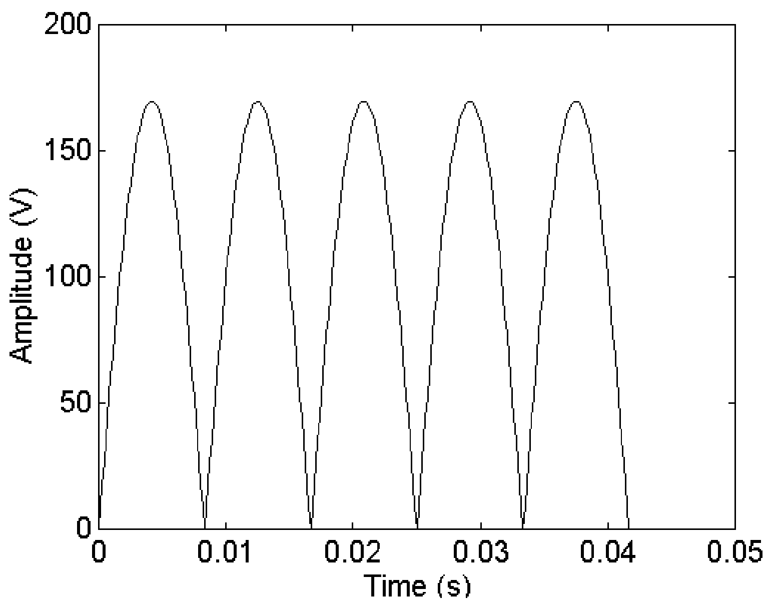

Figure 8.3 Full-wave Rectifier Waveform

Solution

```
diary ex8_2.dat
% generate the full-wave rectifier waveform
f1 = 60;
inv = 1/f1; inc = 1/(80*f1); tnum = 3*inv;
t = 0:inc:tnum;
g1 = 120*sqrt(2)*sin(2*pi*f1*t);
g = abs(g1);
N = length(g);
%
% obtain the exponential Fourier series coefficients

num = 20;
for i = 1:num
    for m = 1:N
     cint(m) = exp(-j*2*pi*(i-1)*m/N)*g(m);
    end
  c(i) = sum(cint)/N;
end
cmag = abs(c);
cphase = angle(c);

%print dc value
disp('dc value of g(t)'); cmag(1)
% plot the magnitude and phase spectrum

f = (0:num-1)*60;
subplot(121), stem(f(1:5),cmag(1:5))
title('Amplitude spectrum')
xlabel('Frequency, Hz')
subplot(122), stem(f(1:5),cphase(1:5))
title('Phase spectrum')
xlabel('Frequency, Hz')
diary
```

dc value of g(t)

```
ans =
        107.5344
```

Figure 8.4 shows the magnitude and phase spectra of Figure 8.3.

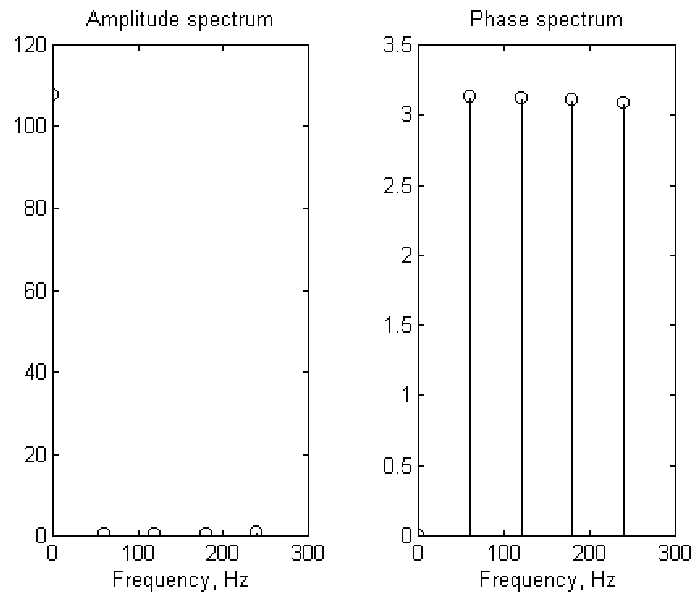

Figure 8.4 Magnitude and Phase Spectra of a Full-wave Rectification Waveform

Example 8.3

The periodic signal shown in Figure 8.5 can be expressed as

$$g(t) = e^{-2t} \qquad -1 \le t < 1$$
$$g(t+2) = g(t)$$

(i) Show that its exponential Fourier series expansion can be expressed as

$$g(t) = \sum_{n=-\infty}^{\infty} \frac{(-1)^n (e^2 - e^{-2})}{2(2 + jn\pi)} \exp(jn\pi t) \tag{8.22}$$

(ii) Using a MATLAB program, synthesize $g(t)$ using 20 terms, i.e.,

$$\hat{g}(t) = \sum_{n=-10}^{10} \frac{(-1)^n (e^2 - e^{-2})}{2(2 + jn\pi)} \exp(jn\pi t)$$

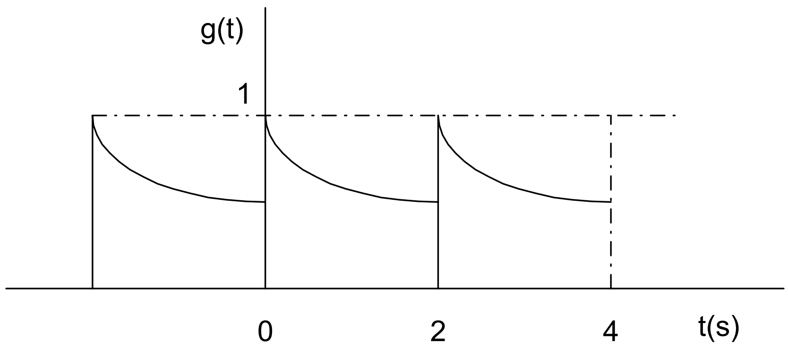

Figure 8.5 Periodic Exponential Signal

Solution

(i)

$$g(t) = \sum_{n=-\infty}^{\infty} c_n \exp(jnw_o t)$$

where

$$c_n = \frac{1}{T_p} \int_{-T_p/2}^{T_p/2} g(t) \exp(-jnw_o t) dt$$

and

$$w_o = \frac{2\pi}{T_p} = \frac{2\pi}{2} = \pi$$

$$c_n = \frac{1}{2} \int_{-1}^{1} \exp(-2t) \exp(-jn\pi t) dt$$

$$c_n = \frac{(-1)^n (e^2 - e^{-2})}{2(2 + jn\pi)}$$

thus

$$g(t) = \sum_{n=-\infty}^{\infty} \frac{(-1)^n (e^2 - e^{-2})}{2(2 + jn\pi)} \exp(jn\pi t)$$

(ii) MATLAB Script

```
% synthesis of g(t) using exponential Fourier series expansion
dt = 0.05;
  tpts = 8.0/dt +1;
cst = exp(2) - exp(-2);

for n = -10:10
  for m = 1:tpts
    g1(n+11,m) = ((0.5*cst*((-1)^n))/(2+j*n*pi))*(exp(j*n*pi*dt*(m-
1)));
  end
end

for m = 1: tpts
 g2 = g1(:,m);
 g3(m) = sum(g2);
end

g = g3';
t = -4:0.05:4.0;
plot(t,g)
xlabel('Time, s')
ylabel('Amplitude')
title('Approximation of g(t)')
```

Figure 8.6 shows the approximation of $g(t)$.

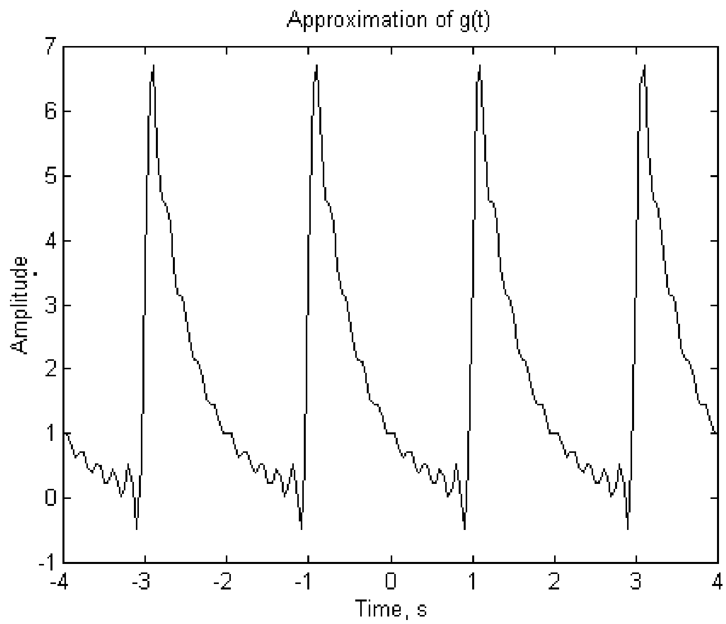

Figure 8.6 An Approximation of $g(t)$.

8.2 FOURIER TRANSFORMS

If $g(t)$ is a nonperiodic deterministic signal expressed as a function of time t, then the Fourier transform of $g(t)$ is given by the integral expression:

$$G(f) = \int_{-\infty}^{\infty} g(t)\exp(-j2\pi ft)dt \qquad (8.23)$$

where

$j = \sqrt{-1}$ and

f denotes frequency

$g(t)$ can be obtained from the Fourier transform $G(f)$ by the Inverse Fourier Transform formula:

$$g(t) = \int_{-\infty}^{\infty} G(f)\exp(j2\pi ft)df \tag{8.24}$$

For a signal $g(t)$ to be Fourier transformable, it should satisfy the Dirichlet's conditions that were discussed in Section 8.1. If $g(t)$ is continuous and non-periodic, then $G(f)$ will be continuous and periodic. However, if *g(t)* is continuous and periodic, then $G(f)$ will discrete and nonperiodic; that is

$$g(t) = g(t \pm nT_p) \tag{8.25}$$

where

T_p = period

then the Fourier transform of $g(t)$ is

$$G(f) = \frac{1}{T_p}\sum_{n=-\infty}^{\infty} c_n \delta(f - \frac{1}{T_p}) \tag{8.26}$$

where

$$c_n = \frac{1}{T_p}\int_{-t_p/2}^{T_p/2} g(t)\exp(-j2\pi n f_o t)dt \tag{8.27}$$

8.2.1 Properties of Fourier transform

If $g(t)$ and $G(f)$ are Fourier transform pairs, and they are expressed as

$$g(t) \Leftrightarrow G(f) \tag{8.28}$$

then the Fourier transform will have the following properties:

<u>Linearity</u>

$$ag_1(t) + bg_2(t) \Leftrightarrow aG_1(f) + bG_2(f) \tag{8.29}$$

where

a and *b* are constants

Time scaling

$$g(at) \Leftrightarrow \frac{1}{|a|} G\left(\frac{f}{a}\right) \tag{8.30}$$

Duality

$$G(t) \Leftrightarrow g(-f) \tag{8.31}$$

Time shifting

$$g(t - t_0) \Leftrightarrow G(f)\exp(-j2\pi f t_0) \tag{8.32}$$

Frequency Shifting

$$\exp(j2 f_C t) g(t) \Leftrightarrow G(f - f_C) \tag{8.33}$$

Definition in the time domain

$$\frac{dg(t)}{dt} \Leftrightarrow j2\pi f G(f) \tag{8.34}$$

Integration in the time domain

$$\int_{-\infty}^{t} g(\tau)d\tau \Leftrightarrow \frac{1}{j2\pi f} G(f) + \frac{G(0)}{2}\delta(f)\,\delta\,(\mathrm{f}) \tag{8.35}$$

Multiplication in the time domain

$$g_1(t)g_2(t) \Leftrightarrow \int_{-\infty}^{\infty} G_1(\lambda)G_2(f - \lambda)d\lambda \tag{8.36}$$

Convolution in the time domain

$$\int_{-\infty}^{\infty} g_1(\tau)g_2(t - \tau)d\tau \Leftrightarrow G_1(f)G_2(f) \tag{8.37}$$

8.3 DISCRETE AND FAST FOURIER TRANSFORMS

Fourier series links a continuous time signal into the discrete-frequency domain. The periodicity of the time-domain signal forces the spectrum to be discrete. The discrete Fourier transform of a discrete-time signal $g[n]$ is given as

$$G[k] = \sum_{n=0}^{N-1} g[n] \exp(-j2\pi nk / N) \quad k = 0,1,\ldots,\text{N-1} \tag{8.38}$$

The inverse discrete Fourier transform, $g[n]$ is

$$g[n] = \sum_{k=0}^{N-1} G[k] \exp(j2\pi nk / N) \quad n = 0,1,\ldots,\text{N-1} \tag{8.39}$$

where

N is the number of time sequence values of $g[n]$. It is also the total number frequency sequence values in $G[k]$.

T is the time interval between two consecutive samples of the input sequence $g[n]$.

F is the frequency interval between two consecutive samples of the output sequence $G[k]$.

N, T, and *F* are related by the expression

$$NT = \frac{1}{F} \tag{8.40}$$

NT is also equal to the record length. The time interval, *T*, between samples should be chosen such that the Shannon's Sampling theorem is satisfied. This means that *T* should be less than the reciprocal of $2f_H$, where f_H is the highest significant frequency component in the continuous time signal $g(t)$ from which the sequence $g[n]$ was obtained. Several fast DFT algorithms require *N* to be an integer power of 2.

A discrete-time function will have a periodic spectrum. In DFT, both the time function and frequency functions are periodic. Because of the periodicity of DFT, it is common to regard points from $n = 1$ through $n = N/2$ as positive, and points from $n = N/2$ through $n = N - 1$ as negative frequencies. In addition, since both the time and frequency sequences are periodic, DFT values at points $n = N/2$ through $n = N - 1$ are equal to the DFT values at points $n = N/2$ through $n = 1$.

In general, if the time-sequence is real-valued, then the DFT will have real components which are even and imaginary components that are odd. Similarly, for an imaginary valued time sequence, the DFT values will have an odd real component and an even imaginary component.

If we define the weighting function W_N as

$$W_N = e^{\frac{-j2\pi}{N}} = e^{-j2\pi FT} \tag{8.41}$$

Equations (8.38) and (8.39) can be re-expressed as

$$G[k] = \sum_{n=0}^{N-1} g[n] W_N^{kn} \tag{8.42}$$

and

$$g[n] = \sum_{k=0}^{N-1} G[k] W_N^{-kn} \tag{8.43}$$

The Fast Fourier Transform, FFT, is an efficient method for computing the discrete Fourier transform. FFT reduces the number of computations needed for computing DFT. For example, if a sequence has N points, and N is an integral power of 2, then DFT requires N^2 operations, whereas FFT requires $\frac{N}{2}\log_2(N)$ complex multiplication, $\frac{N}{2}\log_2(N)$ complex additions and $\frac{N}{2}\log_2(N)$ subtractions. For $N = 1024$, the computational reduction from DFT to FFT is more than 200 to 1.

The FFT can be used to (a) obtain the power spectrum of a signal, (b) do digital filtering, and (c) obtain the correlation between two signals.

8.3.1 MATLAB function fft

The MATLAB function for performing Fast Fourier Transforms is

$$fft(x)$$

where x is the vector to be transformed.

$$fft(x, N)$$

is also MATLAB command that can be used to obtain N-point fft. The vector x is truncated or zeros are added to N, if necessary.

The MATLAB functions for performing inverse fft is

$$ifft(x).$$

$$[z_m, z_p] = fftplot(x, ts)$$

is used to obtain fft and plot the magnitude z_m and z_p of DFT of x. The sampling interval is *ts*. Its default value is 1. The spectra are plotted versus the digital frequency *F*. The following three examples illustrate usage of MATLAB function fft.

Example 8.4

Given the sequence $x[n]$ = (1, 2, 1). (a) Calculate the DFT of $x[n]$. (b) Use the fft algorithm to find DFT of $x[n]$. (c) Compare the results of (a) and (b).

Solution

(a) From Equation (8.42)

$$G[k] = \sum_{n=0}^{N-1} g[n] W_N^{kn}$$

From Equation (8.41)

$$W_3^0 = 1$$

$$W_3^1 = e^{-\frac{j2\pi}{3}} = -0.5 - j0.866$$

$$W_3^2 = e^{-\frac{j4\pi}{3}} = -0.5 + j0.866$$

$$W_3^3 = W_3^0 = 1$$

$$W_3^4 = W_3^1$$

Using Equation (8.41), we have

$$G[0] = \sum_{n=0}^{2} g[n] W_3^0 = 1 + 2 + 1 = 4$$

$$G[1] = \sum_{n=0}^{2} g[n] W_3^n = g[0]W_3^0 + g[1]W_3^1 + g[2]W_3^2$$
$$= 1 + 2(-0.5 - j0.866) + (-0.5 + j0.866) = -0.5 - j0.866$$

$$G[2] = \sum_{n=0}^{2} g[n] W_3^{2n} = g[0]W_3^0 + g[1]W_3^2 + g[2]W_3^4$$
$$= 1 + 2(-0.5 + j0.866) + (-0.5 - j0.866) = -0.5 + j0.866$$

(b) The MATLAB program for performing the DFT of $x[n]$ is

MATLAB Script

```
diary ex8_4.dat
%
x = [1 2 1];
xfft = fft(x)
diary
```

The results are

```
xfft =
       4.0000    -0.5000 - 0.8660i      -0.5000 + 0.8660i
```

(c) It can be seen that the answers obtained from parts (a) and (b) are identical.

Example 8.5

Signal $g(t)$ is given as

$$g(t) = 4e^{-2t}\cos\left[2\pi(10)t\right]u(t)$$

(a) Find the Fourier transform of $g(t)$, i.e., $G(f)$.

(b) Find the DFT of $g(t)$ when the sampling interval is 0.05 s with $N = 1000$.

(c) Find the DFT of $g(t)$ when the sampling interval is 0.2 s with $N = 250$.

(d) Compare the results obtained from parts a, b, and c.

Solution

(a) $g(t)$ can be expressed as

$$g(t) = 4e^{-2t}\left[\frac{1}{2}e^{j20\pi t} + \frac{1}{2}e^{-j20\pi t}\right]u(t)$$

Using the frequency shifting property of the Fourier Transform, we get

$$G(f) = \frac{2}{2 + j2\pi(f - 10)} + \frac{2}{2 + j2\pi(f + 10)}$$

(b, c) The MATLAB program for computing the DFT of $g(t)$ is

MATLAB Script

```
% DFT of g(t)
%  Sample 1, Sampling interval of 0.05 s
ts1 = 0.05;   % sampling interval
fs1 = 1/ts1;  %  Sampling frequency
n1 = 1000;     %  Total Samples
m1 = 1:n1;    %  Number of bins
sint1 = ts1*(m1 - 1);  %  Sampling instants
freq1 = (m1 - 1)*fs1/n1;  % frequencies
gb = (4*exp(-2*sint1)).*cos(2*pi*10*sint1);
gb_abs = abs(fft(gb));
subplot(121)
```

```
plot(freq1, gb_abs)
title('DFT of g(t), 0.05s Sampling interval')
xlabel('Frequency (Hz)')

%  Sample 2,  Sampling interval of 0.2 s
ts2 = 0.2;   % sampling interval
fs2 = 1/ts2;  %  Sampling frequency
n2 = 250;      %  Total Samples
m2 = 1:n2;    %  Number of bins
sint2 = ts2*(m2 - 1);  %  Sampling instants
freq2 = (m2 - 1)*fs2/n2;  % frequencies
gc = (4*exp(-2*sint2)).*cos(2*pi*10*sint2);
gc_abs = abs(fft(gc));
subplot(122)
plot(freq2, gc_abs)
title('DFT of g(t), 0.2s Sampling interval')
xlabel('Frequency (Hz)')
```

The two plots are shown in Figure 8.7.

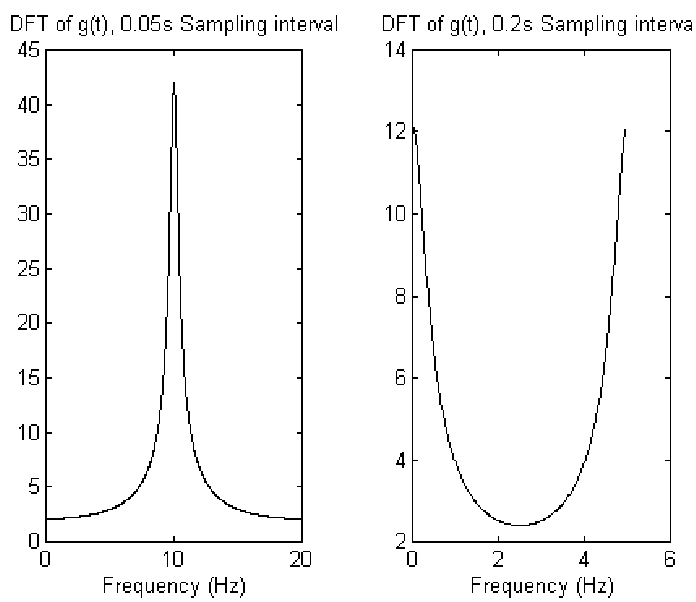

Figure 8.7 DFT of $g(t)$

(d) From Figure 8.7, it can be seen that with the sample interval of 0.05 s, there was no aliasing and spectrum of $G[k]$ in part (b) is almost the same as that of $G(f)$ of part (a). With the sampling interval being 0.2 s (less than the Nyquist rate), there is aliasing and the spectrum of $G[k]$ is different from that of $G(f)$.

Example 8.6

Given a noisy signal

$$g(t) = \sin(2\pi f_1 t) + 0.5n(t)$$

where

$$f_1 = 100 \text{ Hz}$$

n(t) is a normally distributed white noise. The duration of $g(t)$ is 0.5 seconds. Use MATLAB function rand to generate the noise signal. Use MATLAB to obtain the power spectral density of $g(t)$.

Solution

A representative program that can be used to plot the noisy signal and obtain the power spectral density is

MATLAB Script

```
% power spectral estimation of noisy signal
t = 0.0:0.002:0.5;
f1 =100;

% generate the sine portion of signal
x = sin(2*pi*f1*t);

% generate a normally distributed white noise
n = 0.5*randn(size(t));

% generate the noisy signal
y = x+n;
subplot(211), plot(t(1:50),y(1:50)),
title('Nosiy time domain signal')

% power spectral estimation is done
yfft = fft(y,256);
```

```
len = length(yfft);
pyy = yfft.*conj(yfft)/len;
f = (500./256)*(0:127);

subplot(212), plot(f,pyy(1:128)),
title('power spectral density'),
xlabel('frequency in Hz')
```

The plot of the noisy signal and its spectrum is shown in Figure 8.8. The amplitude of the noise and the sinusoidal signal can be changed to observe their effects on the spectrum.

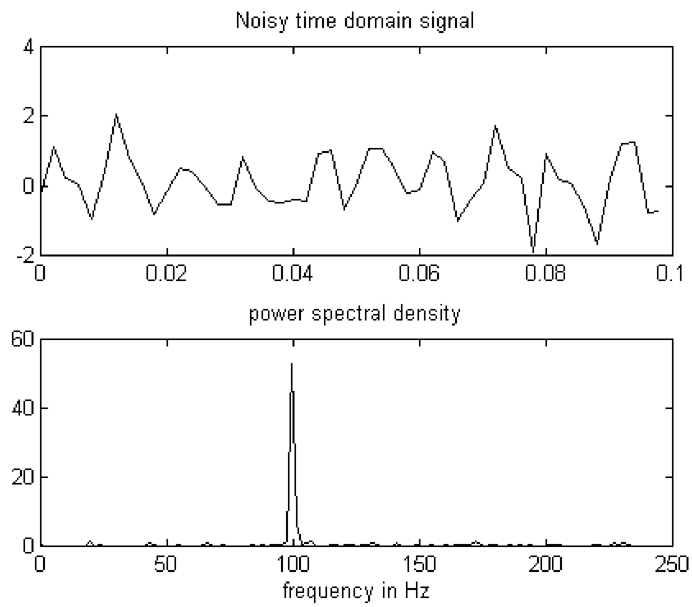

Figure 8.8 Noisy Signal and Its Spectrum

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EXERCISES

8.1 The triangular waveform, shown in Figure P8.1 can be expressed as

$$g(t) = \frac{8A}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{4n^2 - 1} \cos\left((2n-1) w_0 t\right)$$

where

$$w_0 = \frac{1}{T_p}$$

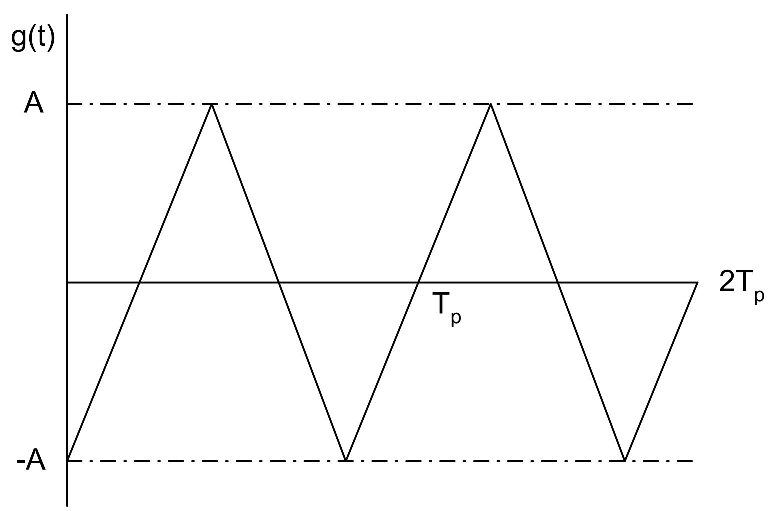

Figure P8.1 Triangular Waveform

If $A = 1$, $T = 8$ ms, and sampling interval is 0.1 ms.

(a) Write MATLAB program to resynthesize $g(t)$ if 20 terms are used.

(b) What is the root-mean-squared value of the function that is the difference between $g(t)$ and the approximation to $g(t)$ when 20 terms are used for the calculation of $g(t)$?

8.2 A periodic pulse train $g(t)$ is shown in Figure P8.2.

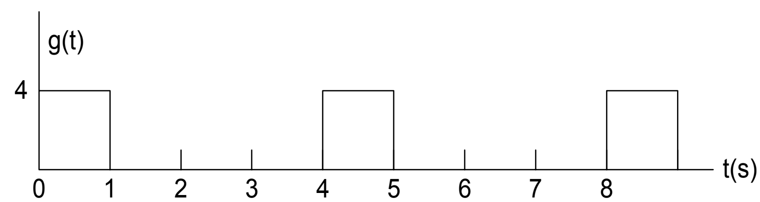

Figure P8.2 Periodic Pulse Train

If $g(t)$ can be expressed by Equation (8.3) ,

(a) Derive expressions for determining the Fourier Series coefficients a_n and b_n .

(b) Write a MATLAB program to obtain a_n and b_n for $n = 0$, 1, ..., 10 by using Equations (8.5) and (8.6).

(c) Resynthesis *g(t)* using 10 terms of the values a_n , b_n obtained from part (b).

8.3 For the half-wave rectifier waveform, shown in Figure P8.3, with a period of 0.01 s and a peak voltage of 17 volts.

(a) Write a MATLAB program to obtain the exponential Fourier series coefficients c_n for $n =$ 0, 1, ..., 20.

(b) Plot the amplitude spectrum.

(c) Using the values obtained in (a), use MATLAB to regenerate the approximation to $g(t)$ when 20 terms of the exponential Fourier series are used.

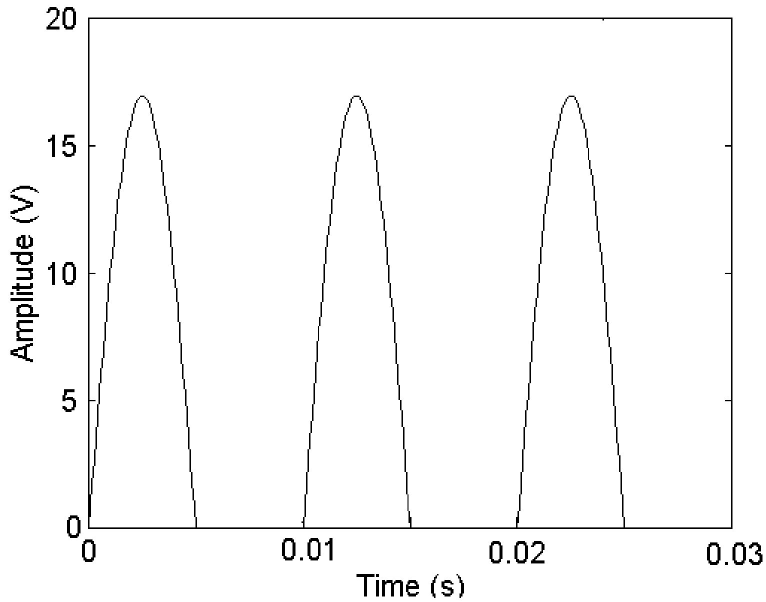

Figure P8.3 Half-Wave Rectifier Waveform

8.4 Figure P8.4(a) is a periodic triangular waveform.

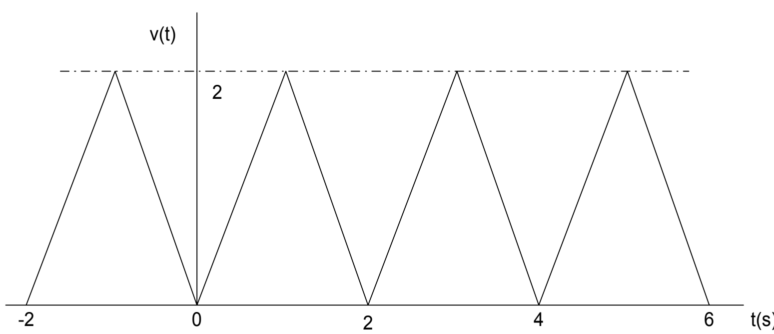

Figure P8.4(a) Periodic Triangular Waveform

(a) Derive the Fourier series coefficients a_n and b_n .

(b) With the signal $v(t)$ of the circuit shown in P8.4(b), derive the expression for the current $i(t)$.

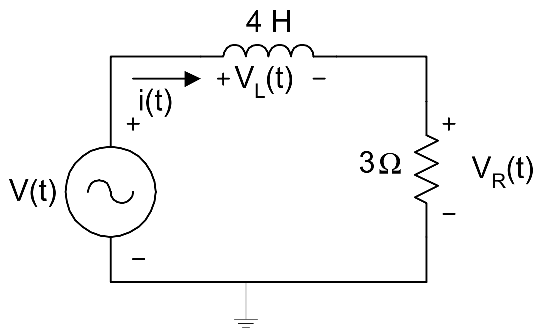

Figure P8.4(b) Simple RL Circuit

(c) Plot the voltages $v_R(t)$, $v_L(t)$ and also the sum of $v_R(t)$ and $v_L(t)$.

(d) Compare the voltages of $v_R(t) + v_L(t)$ to *V(t)*.

8.5 If the periodic waveform shown in Figure 8.5 is the input of the circuit shown in Figure P8.5.

(a) Derive the mathematical expression for $v_C(t)$.

(b) Use MATLAB to plot the signals $g(t)$ and $v_C(t)$.

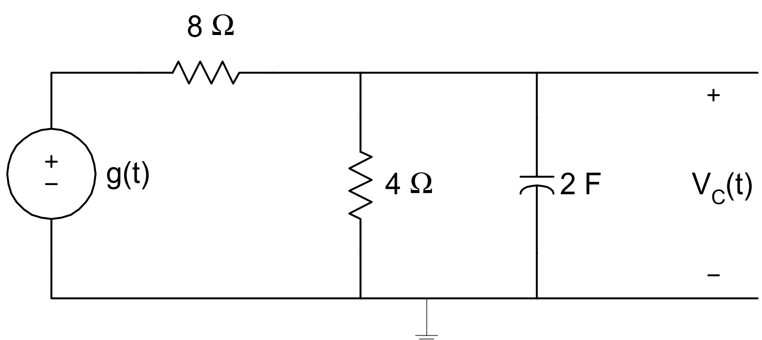

Figure P8.5 RC Circuit

8.6 The unit sample response of a filter is given as

$$h[n] = \begin{pmatrix} 0 & -1 & -1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

(a) Find the discrete Fourier transform of $h[n]$; assume that the values of $h[n]$ not shown are zero.

(b) If the input to the filter is $x[n] = \sin\left[\frac{n}{8}\right]u[n]$, find the output of the filter.

8.7 $g(t) = \sin(200\pi t) + \sin(400\pi t)$

(a) Generate 512 points of $g(t)$. Using the FFT algorithm, generate and plot the frequency content of $g(t)$. Assume a sampling rate of 1200 Hz. Find the power spectrum.

(b) Verify that the frequencies in $g(t)$ are observable in the FFT plot.

8.8 Find the DFT of

$$g(t) = e^{-5t}u(t)$$

(a) Find the Fourier transform of $g(t)$.

(b) Find the DFT of $g(t)$ using the sampling interval of 0.01 s and time duration of 5 seconds.

(c) Compare the results obtained from parts (a) and (b).